

# Package ‘algebraic.mle’

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**Type** Package

**Title** Algebraic Maximum Likelihood Estimators

**Version** 0.9.0

**Maintainer** Alexander Towell <lex@metafunctor.com>

**Description** Defines an algebra over maximum likelihood estimators (MLEs) by providing operators that are closed over MLEs, along with various statistical functions for inference. For background on maximum likelihood estimation, see Casella and Berger (2002, ISBN:978-0534243128). For the delta method and variance estimation, see Lehmann and Casella (1998, ISBN:978-0387985022).

**License** GPL (>= 3)

**Encoding** UTF-8

**ByteCompile** true

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**RoxygenNote** 7.3.3

**URL** <https://github.com/queelius/algebraic.mle>,  
<https://queelius.github.io/algebraic.mle/>

**BugReports** <https://github.com/queelius/algebraic.mle/issues>

**Suggests** rmarkdown, dplyr, knitr, ggplot2, tibble, CDFt

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<code>aic</code>	<i>Generic method for obtaining the AIC of a fitted distribution object fit.</i>
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---

**Description**

Generic method for obtaining the AIC of a fitted distribution object fit.

**Usage**

```
aic(x)
```

**Arguments**

`x` the object to obtain the AIC of

**Value**

The Akaike Information Criterion value (numeric).

---

<code>aic.mle</code>	<i>Method for obtaining the AIC of an 'mle' object.</i>
----------------------	---

---

**Description**

Method for obtaining the AIC of an 'mle' object.

**Usage**

```
## S3 method for class 'mle'
aic(x)
```

**Arguments**

`x` the 'mle' object to obtain the AIC of

**Value**

Numeric AIC value.

---

algebraic.mle	<i>'algebraic.mle': A package for algebraically operating on and generating maximum likelihood estimators from existing maximum likelihood estimators.</i>
---------------	--

---

### Description

The object representing a fitted model is a type of 'mle' object, the maximum likelihood estimator of the model with respect to observed data.

### Details

It has a relatively rich API for working with these objects to help you understand your MLE estimator.#'

### Author(s)

**Maintainer:** Alexander Towell <lex@metafunctor.com> ([ORCID](#))

### See Also

Useful links:

- <https://github.com/queelius/algebraic.mle>
- <https://queelius.github.io/algebraic.mle/>
- Report bugs at <https://github.com/queelius/algebraic.mle/issues>

---

bias	<i>Generic method for computing the bias of an estimator object.</i>
------	--

---

### Description

Generic method for computing the bias of an estimator object.

### Usage

```
bias(x, theta, ...)
```

### Arguments

x	the object to compute the bias of.
theta	true parameter value. usually, this is unknown (NULL), in which case we estimate the bias
...	pass additional arguments

### Value

The bias of the estimator. The return type depends on the specific method.

---

bias.mle	<i>Computes the bias of an 'mle' object assuming the large sample approximation is valid and the MLE regularity conditions are satisfied. In this case, the bias is zero (or zero vector).</i>
----------	--

---

**Description**

This is not a good estimate of the bias in general, but it's arguably better than returning 'NULL'.

**Usage**

```
## S3 method for class 'mle'
bias(x, theta = NULL, ...)
```

**Arguments**

x	the 'mle' object to compute the bias of.
theta	true parameter value. normally, unknown (NULL), in which case we estimate the bias (say, using bootstrap)
...	additional arguments to pass

**Value**

Numeric vector of zeros (asymptotic bias is zero under regularity conditions).

---

bias.mle_boot	<i>Computes the estimate of the bias of a 'mle_boot' object.</i>
---------------	--

---

**Description**

Computes the estimate of the bias of a 'mle\_boot' object.

**Usage**

```
## S3 method for class 'mle_boot'
bias(x, theta = NULL, ...)
```

**Arguments**

x	the 'mle_boot' object to compute the bias of.
theta	true parameter value (not used for 'mle_boot').
...	pass additional arguments (not used)

**Value**

Numeric vector of estimated bias (mean of bootstrap replicates minus original estimate).

---

confint.mle	<i>Function to compute the confidence intervals of 'mle' objects.</i>
-------------	---

---

### Description

Function to compute the confidence intervals of 'mle' objects.

### Usage

```
## S3 method for class 'mle'
confint(object, parm = NULL, level = 0.95, use_t_dist = FALSE, ...)
```

### Arguments

object	the 'mle' object to compute the confidence intervals for
parm	the parameters to compute the confidence intervals for (not used)
level	confidence level, defaults to 0.95 (alpha=.05)
use_t_dist	logical, whether to use the t-distribution to compute the confidence intervals.
...	additional arguments to pass

### Value

Matrix of confidence intervals with columns for lower and upper bounds.

---

confint.mle_boot	<i>Method for obtained the confidence interval of an 'mle_boot' object. Note: This implements the 'vcov' method defined in 'stats'.</i>
------------------	---

---

### Description

Method for obtained the confidence interval of an 'mle\_boot' object. Note: This implements the 'vcov' method defined in 'stats'.

### Usage

```
## S3 method for class 'mle_boot'
confint(
  object,
  parm = NULL,
  level = 0.95,
  type = c("norm", "basic", "perc", "bca"),
  ...
)
```

**Arguments**

object	the 'mle_boot' object to obtain the confidence interval of
parm	the parameter to obtain the confidence interval of (not used)
level	the confidence level
type	the type of confidence interval to compute
...	additional arguments to pass into 'boot.ci'

**Value**

Matrix of bootstrap confidence intervals with columns for lower and upper bounds.

---

confint_from_sigma	<i>Function to compute the confidence intervals from a variance-covariance matrix</i>
--------------------	---

---

**Description**

Function to compute the confidence intervals from a variance-covariance matrix

**Usage**

```
confint_from_sigma(sigma, theta, level = 0.95)
```

**Arguments**

sigma	either the variance-covariance matrix or the vector of variances of the parameter estimator
theta	the point estimate
level	confidence level, defaults to 0.95 (alpha=.05)

**Value**

Matrix of confidence intervals with rows for each parameter and columns for lower and upper bounds.

**Examples**

```
# Compute CI for a bivariate parameter
theta <- c(mu = 5.2, sigma2 = 4.1)
vcov_matrix <- diag(c(0.1, 0.5)) # Variance of estimators

confint_from_sigma(vcov_matrix, theta)
confint_from_sigma(vcov_matrix, theta, level = 0.99)
```

---

expectation.mle	<i>Expectation operator applied to 'x' of type 'mle' with respect to a function 'g'. That is, 'E(g(x))'.</i>
-----------------	--

---

### Description

Optionally, we use the CLT to construct a CI('alpha') for the estimate of the expectation. That is, we estimate 'E(g(x))' with the sample mean and Var(g(x)) with the  $\sigma^2/n$ , where  $\sigma^2$  is the sample variance of g(x) and n is the number of samples. From these, we construct the CI.

### Usage

```
## S3 method for class 'mle'
expectation(x, g = function(t) t, ..., control = list())
```

### Arguments

x	'mle' object
g	characteristic function of interest, defaults to identity
...	additional arguments to pass to 'g'
control	a list of control parameters: compute_stats - Whether to compute CIs for the expectations, defaults to FALSE n - The number of samples to use for the MC estimate, defaults to 10000 alpha - The significance level for the confidence interval, defaults to 0.05

### Value

If 'compute\_stats' is FALSE, then the estimate of the expectation, otherwise a list with the following components: value - The estimate of the expectation ci - The confidence intervals for each component of the expectation n - The number of samples

---

is_mle	<i>Determine if an object 'x' is an 'mle' object.</i>
--------	---

---

### Description

Determine if an object 'x' is an 'mle' object.

### Usage

```
is_mle(x)
```

### Arguments

x	the object to test
---	--------------------



**Value**

Logical TRUE if x is an mle object, FALSE otherwise.

**Examples**

```
fit <- mle(theta.hat = c(mu = 5), sigma = matrix(0.1))
is_mle(fit)      # TRUE
is_mle(list(a=1)) # FALSE
```

---

is\_mle\_boot

*Determine if an object is an 'mle\_boot' object.*

---

**Description**

Determine if an object is an 'mle\_boot' object.

**Usage**

```
is_mle_boot(x)
```

**Arguments**

x                    the object to test

**Value**

Logical TRUE if x is an mle\_boot object, FALSE otherwise.

**Examples**

```
# Create a simple mle object (not bootstrap)
fit_mle <- mle(theta.hat = 5, sigma = matrix(0.1))
is_mle_boot(fit_mle) # FALSE

# Bootstrap example would return TRUE
```

---

loglik_val	<i>Generic method for obtaining the log-likelihood value of a fitted MLE object.</i>
------------	--

---

**Description**

Generic method for obtaining the log-likelihood value of a fitted MLE object.

**Usage**

```
loglik_val(x, ...)
```

**Arguments**

x	the object to obtain the log-likelihood of
...	additional arguments to pass

**Value**

The log-likelihood value (numeric).

---

loglik_val.mle	<i>Method for obtaining the log-likelihood of an 'mle' object.</i>
----------------	--

---

**Description**

Method for obtaining the log-likelihood of an 'mle' object.

**Usage**

```
## S3 method for class 'mle'  
loglik_val(x, ...)
```

**Arguments**

x	the log-likelihood 'l' evaluated at 'x', 'l(x)'. ... additional arguments to pass
---	--

**Value**

the log-likelihood of the fitted mle object 'x'

---

marginal.mle	<i>Method for obtaining the marginal distribution of an MLE that is based on asymptotic assumptions:</i>
--------------	--

---

**Description**

`'x ~ MVN(params(x), inv(H)(x))'`

**Usage**

```
## S3 method for class 'mle'
marginal(x, indices)
```

**Arguments**

x	The distribution object.
indices	The indices of the marginal distribution to obtain.

**Details**

where H is the (observed or expectation) Fisher information matrix.

**Value**

An mle object representing the marginal distribution for the selected parameter indices.

---

mle	<i>Constructor for making 'mle' objects, which provides a common interface for maximum likelihood estimators.</i>
-----	---

---

**Description**

This MLE makes the asymptotic assumption by default. Other MLEs, like `'mle_boot'`, may not make this assumption.

**Usage**

```
mle(
  theta.hat,
  loglike = NULL,
  score = NULL,
  sigma = NULL,
  info = NULL,
  obs = NULL,
  nobs = NULL,
  superclasses = NULL
)
```

**Arguments**

theta.hat	the MLE
loglike	the log-likelihood of 'theta.hat' given the data
score	the score function evaluated at 'theta.hat'
sigma	the variance-covariance matrix of 'theta.hat' given that data
info	the information matrix of 'theta.hat' given the data
obs	observation (sample) data
nobs	number of observations in 'obs'
superclasses	class (or classes) with 'mle' as base

**Value**

An object of class mle.

**Examples**

```
# MLE for normal distribution (mean and variance)
set.seed(123)
x <- rnorm(100, mean = 5, sd = 2)
n <- length(x)
mu_hat <- mean(x)
var_hat <- mean((x - mu_hat)^2) # MLE of variance

# Asymptotic variance-covariance of MLE
# For normal: Var(mu_hat) = sigma^2/n, Var(var_hat) = 2*sigma^4/n
sigma_matrix <- diag(c(var_hat/n, 2*var_hat^2/n))

fit <- mle(
  theta.hat = c(mu = mu_hat, var = var_hat),
  sigma = sigma_matrix,
  loglike = sum(dnorm(x, mu_hat, sqrt(var_hat), log = TRUE)),
  nobs = n
)

params(fit)
vcov(fit)
confint(fit)
```

---

mle\_boot

*Bootstrapped MLE*


---

**Description**

Sometimes, the large sample asymptotic theory of MLEs is not applicable. In such cases, we can use the bootstrap to estimate the sampling distribution of the MLE.

**Usage**

```
mle_boot(x)
```

**Arguments**

x                    the 'boot' return value

**Details**

This takes an approach similiar to the 'mle\_numerical' object, which is a wrapper for a 'stats::optim' return value, or something that is compatible with the 'optim' return value. Here, we take a 'boot' object, which is the sampling distribution of an MLE, and wrap it in an 'mle\_boot' object and then provide a number of methods for the 'mle\_boot' object that satisfies the concept of an 'mle' object. Look up the 'boot' package for more information on the bootstrap.

**Value**

An mle\_boot object (wrapper for boot object).

**Examples**

```
# Bootstrap MLE for mean of exponential distribution
set.seed(123)
x <- rexp(50, rate = 2)

# Statistic function: MLE of rate parameter
rate_mle <- function(data, indices) {
  d <- data[indices]
  1 / mean(d) # MLE of rate is 1/mean
}

# Run bootstrap
boot_result <- boot::boot(data = x, statistic = rate_mle, R = 200)

# Wrap in mle_boot
fit <- mle_boot(boot_result)
params(fit)
bias(fit)
confint(fit)
```

---

mle_numerical	<i>This function takes the output of 'optim', 'newton_raphson', or 'sim_anneal' and turns it into an 'mle_numerical' (subclass of 'mle') object.</i>
---------------	--

---

**Description**

This function takes the output of 'optim', 'newton\_raphson', or 'sim\_anneal' and turns it into an 'mle\_numerical' (subclass of 'mle') object.

**Usage**

```
mle_numerical(sol, options = list(), superclasses = NULL)
```

**Arguments**

```
sol           the output of 'optim' or 'newton_raphson'
options       list, options for things like sigma and FIM
superclasses  list, superclasses to add to the 'mle_numerical' object
```

**Value**

An object of class `mle_numerical` (subclass of `mle`).

**Examples**

```
# Fit exponential distribution using optim
set.seed(123)
x <- rexp(100, rate = 2)

# Log-likelihood for exponential distribution
loglik <- function(rate) {
  if (rate <= 0) return(-Inf)
  sum(dexp(x, rate = rate, log = TRUE))
}

# Optimize (maximize by setting fnscale = -1)
result <- optim(
  par = 1,
  fn = loglik,
  method = "Brent",
  lower = 0.01, upper = 10,
  hessian = TRUE,
  control = list(fnscale = -1)
)

# Wrap in mle_numerical
fit <- mle_numerical(result, options = list(nobs = length(x)))
params(fit)
se(fit)
```

---

`mle_weighted`

*Accepts a list of 'mle' objects for some parameter, say 'theta', and combines them into a single estimator 'mle\_weighted'.*

---

**Description**

It combines the 'mle' objects by adding them together, weighted by the inverse of their respective variance-covariance matrix (information matrix). Intuitively, the higher the variance, the less weight an 'mle' is given in the summation, or alternatively, the more information it has about the parameter, the more weight it is given in the summation.

**Usage**

```
mle_weighted(mles)
```

**Arguments**

`mles` A list of 'mle' objects, all for the same parameter.

**Details**

Each 'mle' object should have an 'observed\_fim' method, which returns the Fisher information matrix (FIM) for the parameter. The FIM is assumed to be the negative of the expected value of the Hessian of the log-likelihood function. The 'mle' objects should also have a 'params' method, which returns the parameter vector.

We assume that the observations used to estimate each of the MLE objects in 'mles' are independent.

**Value**

An object of type `mle_weighted` (which inherits from `mle`) that is the weighted sum of the `mle` objects.

**Examples**

```
# Combine three independent estimates of mean
set.seed(123)

# Three independent samples
x1 <- rnorm(50, mean = 10, sd = 2)
x2 <- rnorm(30, mean = 10, sd = 2)
x3 <- rnorm(70, mean = 10, sd = 2)

# Create MLE objects for each sample
make_mean_mle <- function(x) {
  n <- length(x)
  s2 <- var(x)
  mle(theta.hat = mean(x),
      sigma = matrix(s2/n),
      info = matrix(n/s2),
      nobs = n)
}

fit1 <- make_mean_mle(x1)
fit2 <- make_mean_mle(x2)
fit3 <- make_mean_mle(x3)

# Combine using inverse-variance weighting
combined <- mle_weighted(list(fit1, fit2, fit3))
params(combined)
se(combined)
```

---

mse	<i>Generic method for computing the mean squared error (MSE) of an estimator, 'mse(x) = E[(x-mu)^2]' where 'mu' is the true parameter value.</i>
-----	--

---

**Description**

Generic method for computing the mean squared error (MSE) of an estimator, 'mse(x) = E[(x-mu)^2]' where 'mu' is the true parameter value.

**Usage**

```
mse(x, theta)
```

**Arguments**

x	the object to compute the MSE of
theta	the true parameter value

**Value**

The mean squared error (matrix or scalar).

---

mse.mle	<i>Computes the MSE of an 'mle' object.</i>
---------	---

---

**Description**

The MSE of an estimator is just the expected sum of squared differences, e.g., if the true parameter value is 'x' and we have an estimator 'x.hat', then the MSE is `“mse(x.hat) = E[(x.hat-x) vcov(x.hat) + bias(x.hat, x)]“`

**Usage**

```
## S3 method for class 'mle'
mse(x, theta = NULL)
```

**Arguments**

x	the 'mle' object to compute the MSE of.
theta	true parameter value, defaults to 'NULL' for unknown. If 'NULL', then we let the bias method deal with it. Maybe it has a nice way of estimating the bias.



**Details**

Since 'x' is not typically known, we normally must estimate the bias. Asymptotically, assuming the regularity conditions, the bias of an MLE is zero, so we can estimate the MSE as 'mse(x.hat) = vcov(x.hat)', but for small samples, this is not generally the case. If we can estimate the bias, then we can replace the bias with an estimate of the bias.

Sometimes, we can estimate the bias analytically, but if not, we can use something like the bootstrap. For example, if we have a sample of size 'n', we can bootstrap the bias by sampling 'n' observations with replacement, computing the MLE, and then computing the difference between the bootstrapped MLE and the MLE. We can repeat this process 'B' times, and then average the differences to get an estimate of the bias.

**Value**

The MSE as a scalar (univariate) or matrix (multivariate).

---

mse.mle_boot	<i>Computes the estimate of the MSE of a 'boot' object.</i>
--------------	---

---

**Description**

Computes the estimate of the MSE of a 'boot' object.

**Usage**

```
## S3 method for class 'mle_boot'
mse(x, theta = NULL, ...)
```

**Arguments**

x	the 'boot' object to compute the MSE of.
theta	true parameter value (not used for 'mle_boot')
...	pass additional arguments into 'vcov'

**Value**

The MSE matrix estimated from bootstrap variance and bias.

---

nobs.mle	<i>Method for obtaining the number of observations in the sample used by an 'mle'.</i>
----------	--

---

**Description**

Method for obtaining the number of observations in the sample used by an 'mle'.

**Usage**

```
## S3 method for class 'mle'
nobs(object, ...)
```

**Arguments**

object	the 'mle' object to obtain the number of observations for
...	additional arguments to pass (not used)

**Value**

Integer number of observations, or NULL if not available.

---

nobs.mle_boot	<i>Method for obtaining the number of observations in the sample used by an 'mle'.</i>
---------------	--

---

**Description**

Method for obtaining the number of observations in the sample used by an 'mle'.

**Usage**

```
## S3 method for class 'mle_boot'
nobs(object, ...)
```

**Arguments**

object	the 'mle' object to obtain the number of observations for
...	additional arguments to pass (not used)

**Value**

Integer number of observations in the original sample.

---

nparams.mle	<i>Method for obtaining the number of parameters of an 'mle' object.</i>
-------------	--

---

**Description**

Method for obtaining the number of parameters of an 'mle' object.

**Usage**

```
## S3 method for class 'mle'  
nparams(x)
```

**Arguments**

x                    the 'mle' object to obtain the number of parameters of

**Value**

Integer number of parameters.

---

nparams.mle_boot	<i>Method for obtaining the number of parameters of an 'boot' object.</i>
------------------	---

---

**Description**

Method for obtaining the number of parameters of an 'boot' object.

**Usage**

```
## S3 method for class 'mle_boot'  
nparams(x)
```

**Arguments**

x                    the 'boot' object to obtain the number of parameters of

**Value**

Integer number of parameters.

---

obs.mle	<i>Method for obtaining the observations used by the 'mle' object 'x'.</i>
---------	--

---

**Description**

Method for obtaining the observations used by the 'mle' object 'x'.

**Usage**

```
## S3 method for class 'mle'  
obs(x)
```

**Arguments**

x                    the 'mle' object to obtain the number of observations for

**Value**

The observation data used to fit the MLE, or NULL if not stored.

---

obs.mle_boot	<i>Method for obtaining the observations used by the 'mle'.</i>
--------------	---

---

**Description**

Method for obtaining the observations used by the 'mle'.

**Usage**

```
## S3 method for class 'mle_boot'  
obs(x)
```

**Arguments**

x                    the 'mle' object to obtain the number of observations for

**Value**

The original data used for bootstrapping.

---

observed_fim	<i>Generic method for computing the observed FIM of an 'mle' object.</i>
--------------	--

---

**Description**

Fisher information is a way of measuring the amount of information that an observable random variable 'X' carries about an unknown parameter 'theta' upon which the probability of 'X' depends.

**Usage**

```
observed_fim(x, ...)
```

**Arguments**

x	the object to obtain the fisher information of
...	additional arguments to pass

**Details**

The inverse of the Fisher information matrix is the variance-covariance of the MLE for 'theta'. Some MLE objects do not have an observed FIM, e.g., if the MLE's sampling distribution was bootstrapped.

**Value**

The observed Fisher Information Matrix.

---

observed_fim.mle	<i>Function for obtaining the observed FIM of an 'mle' object.</i>
------------------	--

---

**Description**

Function for obtaining the observed FIM of an 'mle' object.

**Usage**

```
## S3 method for class 'mle'
observed_fim(x, ...)
```

**Arguments**

x	the 'mle' object to obtain the FIM of.
...	pass additional arguments

**Value**

The observed Fisher Information Matrix, or NULL if not available.

---

orthogonal	<i>Generic method for determining the orthogonal parameters of an estimator.</i>
------------	--

---

**Description**

Generic method for determining the orthogonal parameters of an estimator.

**Usage**

```
orthogonal(x, tol, ...)
```

**Arguments**

x	the estimator
tol	the tolerance for determining if a number is close enough to zero
...	additional arguments to pass

**Value**

Logical vector or matrix indicating which parameters are orthogonal.

---

orthogonal.mle	<i>Method for determining the orthogonal components of an 'mle' object 'x'.</i>
----------------	---

---

**Description**

Method for determining the orthogonal components of an 'mle' object 'x'.

**Usage**

```
## S3 method for class 'mle'
orthogonal(x, tol = sqrt(.Machine$double.eps), ...)
```

**Arguments**

x	the 'mle' object
tol	the tolerance for determining if a number is close enough to zero
...	pass additional arguments

**Value**

Logical matrix indicating which off-diagonal FIM elements are approximately zero (orthogonal parameters), or NULL if FIM unavailable.

---

params.mle	<i>Method for obtaining the parameters of an 'mle' object.</i>
------------	--

---

**Description**

Method for obtaining the parameters of an 'mle' object.

**Usage**

```
## S3 method for class 'mle'  
params(x)
```

**Arguments**

x                    the 'mle' object to obtain the parameters of

**Value**

Numeric vector of parameter estimates.

---

params.mle_boot	<i>Method for obtaining the parameters of an 'boot' object.</i>
-----------------	---

---

**Description**

Method for obtaining the parameters of an 'boot' object.

**Usage**

```
## S3 method for class 'mle_boot'  
params(x)
```

**Arguments**

x                    the 'boot' object to obtain the parameters of.

**Value**

Numeric vector of parameter estimates (the original MLE).

---

pred	<i>Generic method for computing the predictive confidence interval given an estimator object 'x'.</i>
------	---

---

**Description**

Generic method for computing the predictive confidence interval given an estimator object 'x'.

**Usage**

```
pred(x, samp = NULL, alpha = 0.05, ...)
```

**Arguments**

x	the estimator object
samp	a sampler for random variable that is parameterized by mle 'x'
alpha	(1-alpha)/2 confidence interval
...	additional arguments to pass

**Value**

Matrix of predictive confidence intervals.

---

pred.mle	<i>Estimate of predictive interval of 'T\data' using Monte Carlo integration.</i>
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---

**Description**

Let  $T|x \sim f(t|x)$  be the pdf of vector 'T' given MLE 'x' and  $x \sim MVN(\text{params}(x), \text{vcov}(x))$  be the estimate of the sampling distribution of the MLE for the parameters of 'T'. Then,  $(T,x) \sim f(t,x) = f(t|x) f(x)$  is the joint distribution of '(T,x)'. To find 'f(t)' for a fixed 't', we integrate 'f(t,x)' over 'x' using Monte Carlo integration to find the marginal distribution of 'T'. That is, we:

**Usage**

```
## S3 method for class 'mle'
pred(x, samp, alpha = 0.05, R = 50000, ...)
```

**Arguments**

x	an 'mle' object.
samp	The sampler for the distribution that is parameterized by the MLE 'x', i.e., 'T x'.
alpha	(1-alpha)-predictive interval for 'T x'. Defaults to 0.05.
R	number of samples to draw from the sampling distribution of 'x'. Defaults to 50000.
...	additional arguments to pass into 'samp'.



**Details**

1. Sample from MVN 'x' 2. Compute 'f(t,x)' for each sample 3. Take the mean of the 'f(t,x)' values as an estimate of 'f(t)'.

The 'samp' function is used to sample from the distribution of 'T|x'. It should be designed to take

**Value**

Matrix with columns for mean, lower, and upper bounds of the predictive interval.

---

print.mle	<i>Print method for 'mle' objects.</i>
-----------	--

---

**Description**

Print method for 'mle' objects.

**Usage**

```
## S3 method for class 'mle'
print(x, ...)
```

**Arguments**

x	the 'mle' object to print
...	additional arguments to pass

**Value**

Invisibly returns x.

---

print.summary_mle	<i>Function for printing a 'summary' object for an 'mle' object.</i>
-------------------	--

---

**Description**

Function for printing a 'summary' object for an 'mle' object.

**Usage**

```
## S3 method for class 'summary_mle'
print(x, ...)
```

**Arguments**

x	the 'summary_mle' object
...	pass additional arguments

**Value**

Invisibly returns `x`.

---

`rmap.mle`

*Computes the distribution of 'g(x)' where 'x' is an 'mle' object.*

---

**Description**

By the invariance property of the MLE, if 'x' is an 'mle' object, then under the right conditions, asymptotically, 'g(x)' is normally distributed,  $g(x) \sim \text{normal}(g(\text{point}(x)), \text{sigma})$  where 'sigma' is the variance-covariance of 'f(x)'

**Usage**

```
## S3 method for class 'mle'
rmap(x, g, ..., n = 1000L, method = c("mc", "delta"))
```

**Arguments**

<code>x</code>	an 'mle' object
<code>g</code>	a function
<code>...</code>	additional arguments to pass to the 'g' function
<code>n</code>	number of samples to take to estimate distribution of 'g(x)' if 'method == "mc"'.
<code>method</code>	method to use to estimate distribution of 'g(x)', "delta" or "mc".

**Details**

We provide two different methods for estimating the variance-covariance of 'f(x)': `method = "delta"` -> delta method `method = "mc"` -> monte carlo method

**Value**

An `mle` object of class `rmap_mle` representing the transformed MLE with variance estimated by the specified method.

**Examples**

```
# MLE for normal distribution
set.seed(123)
x <- rnorm(100, mean = 5, sd = 2)
n <- length(x)
fit <- mle(
  theta.hat = c(mu = mean(x), var = var(x)),
  sigma = diag(c(var(x)/n, 2*var(x)^2/n)),
  nobs = n
)
```

```
# Transform: compute MLE of standard deviation (sqrt of variance)
# Using delta method
g <- function(theta) sqrt(theta[2])
sd_mle <- rmap(fit, g, method = "delta")
params(sd_mle)
se(sd_mle)
```

---

sampler.mle

*Method for sampling from an 'mle' object.*


---

### Description

It creates a sampler for the 'mle' object. It returns a function that accepts a single parameter 'n' denoting the number of samples to draw from the 'mle' object.

### Usage

```
## S3 method for class 'mle'
sampler(x, ...)
```

### Arguments

x                    the 'mle' object to create sampler for  
...                    additional arguments to pass

### Value

A function that takes parameter n and returns n samples from the asymptotic distribution of the MLE.

---

sampler.mle\_boot

*Method for sampling from an 'mle\_boot' object.*


---

### Description

It creates a sampler for the 'mle\_boot' object. It returns a function that accepts a single parameter 'n' denoting the number of samples to draw from the 'mle\_boot' object.

### Usage

```
## S3 method for class 'mle_boot'
sampler(x, ...)
```

### Arguments

x                    the 'mle\_boot' object to create sampler for  
...                    additional arguments to pass (not used)

**Details**

Unlike the ‘sampler’ method for the more general ‘mle’ objects, for ‘mle\_boot’ objects, we sample from the bootstrap replicates, which are more representative of the sampling distribution, particularly for small samples.

**Value**

A function that takes parameter n and returns n samples drawn from the bootstrap replicates.

---

score_val	<i>Generic method for computing the score of an estimator object (gradient of its log-likelihood function evaluated at the MLE).</i>
-----------	--

---

**Description**

Generic method for computing the score of an estimator object (gradient of its log-likelihood function evaluated at the MLE).

**Usage**

```
score_val(x, ...)
```

**Arguments**

x	the object to compute the score of.
...	pass additional arguments

**Value**

The score vector evaluated at the MLE.

---

score_val.mle	<i>Computes the score of an ‘mle’ object (score evaluated at the MLE).</i>
---------------	--

---

**Description**

If regularity conditions are satisfied, it should be zero (or approximately, if rounding errors occur).

**Usage**

```
## S3 method for class 'mle'
score_val(x, ...)
```

**Arguments**

x                    the 'mle' object to compute the score of.  
 ...                  additional arguments to pass (not used)

**Value**

The score vector evaluated at the MLE, or NULL if not available.

---

se	<i>Generic method for obtaining the standard errors of an estimator.</i>
----	--

---

**Description**

Generic method for obtaining the standard errors of an estimator.

**Usage**

```
se(x, ...)
```

**Arguments**

x                    the estimator  
 ...                  additional arguments to pass

**Value**

Vector of standard errors for each parameter.

---

se.mle	<i>Function for obtaining an estimate of the standard error of the MLE object 'x'.</i>
--------	--

---

**Description**

Function for obtaining an estimate of the standard error of the MLE object 'x'.

**Usage**

```
## S3 method for class 'mle'
se(x, se.matrix = FALSE, ...)
```

**Arguments**

x                    the MLE object  
 se.matrix          if 'TRUE', return the square root of the variance-covariance  
 ...                  additional arguments to pass (not used)

**Value**

Vector of standard errors, or matrix if `se.matrix = TRUE`, or `NULL` if variance-covariance is not available.

---

<code>summary.mle</code>	<i>Function for obtaining a summary of 'object', which is a fitted 'mle' object.</i>
--------------------------	--

---

**Description**

Function for obtaining a summary of 'object', which is a fitted 'mle' object.

**Usage**

```
## S3 method for class 'mle'
summary(object, ...)
```

**Arguments**

<code>object</code>	the 'mle' object
<code>...</code>	pass additional arguments

**Value**

An object of class `summary_mle`.

---

<code>vcov.mle</code>	<i>Computes the variance-covariance matrix of 'mle' object.</i>
-----------------------	---

---

**Description**

Computes the variance-covariance matrix of 'mle' object.

**Usage**

```
## S3 method for class 'mle'
vcov(object, ...)
```

**Arguments**

<code>object</code>	the 'mle' object to obtain the variance-covariance of
<code>...</code>	additional arguments to pass (not used)

**Value**

the variance-covariance matrix

---

vcov.mle_boot	<i>Computes the variance-covariance matrix of 'boot' object. Note: This implements the 'vcov' method defined in 'stats'.</i>
---------------	--

---

**Description**

Computes the variance-covariance matrix of 'boot' object. Note: This implements the 'vcov' method defined in 'stats'.

**Usage**

```
## S3 method for class 'mle_boot'  
vcov(object, ...)
```

**Arguments**

object	the 'boot' object to obtain the variance-covariance of
...	additional arguments to pass into 'stats::cov'

**Value**

The variance-covariance matrix estimated from bootstrap replicates.

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