

Package ‘gaussratiovegind’

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Title Distribution of Gaussian Ratios

Version 2.0.1

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Description It is well known that the distribution of a Gaussian ratio does not follow a Gaussian distribution.
The lack of awareness among users of vegetation indices about this non-Gaussian nature could lead to incorrect statistical modeling and interpretation.
This package provides tools to accurately handle and analyse such ratios: density function, parameter estimation, simulation.
An example on the study of chlorophyll fluorescence can be found in A. El Ghaziri et al. (2023) <[doi:10.3390/rs15020528](https://doi.org/10.3390/rs15020528)> and another method for parameter estimation is given in Bouhlel et al. (2023) <[doi:10.23919/EUSIPCO58844.2023.10290111](https://doi.org/10.23919/EUSIPCO58844.2023.10290111)>.

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URL <https://forgemia.inra.fr/imhorphen/gaussratiovegind>

BugReports <https://forgemia.inra.fr/imhorphen/gaussratiovegind/-/issues>

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arabidopsis	<i>Statistics on Chlorophyll Fluorescence Parameters</i>
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Description

Mean and standard deviation values on healthy and diseased tissues of chlorophyll fluorescence parameters F_0 (minimum fluorescence) and F_m (maximum fluorescence) for a dataset of *Arabidopsis thaliana* plants infected with fungal pathogen data; parameters of the distribution of the ratio

$$\frac{F_v}{F_m} = \frac{F_m - F_0}{F_m}.$$

Usage

arabidopsis

Format

A data frame with 10 rows and 6 columns:

time times of the acquisition of chlorophyll fluorescence images

condition indicates if the plant was inoculated: healthy (inoculated with water) or diseased (inoculated with the pathogen)

mF0, sF0 Mean and standard deviation values of the chlorophyll parameter F_0

mFm, sFm Mean and standard deviation values of the chlorophyll parameter F_m

beta, rho, delta the β , ρ and δ_y parameters of the distribution of $\frac{F_v}{F_m} = \frac{F_m - F_0}{F_m}$ (distributed according to a normal ratio distribution, see Details)

Details

On each leaf picture, the F_0 and F_m values are normally distributed. Hence, $\frac{F_0}{F_m}$ is a ratio of two normal distributions.

Let μ_{F_0} and σ_{F_0} the mean and standard deviation of F_0 and μ_{F_m} and σ_{F_m} the mean and standard deviation of F_m . The parameters β , ρ and δ_y are given by:

$$\beta = \frac{\mu_{F_0}}{\mu_{F_m}}$$

$$\rho = \frac{\sigma_{F_m}}{\sigma_{F_0}}$$

$$\delta_y = \frac{\sigma_{F_m}}{\mu_{F_m}}$$

References

- El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). doi:10.3390/rs15020528
- Pavicic, M., Overmyer, K., Rehman, A.u., Jones, P., Jacobson, D., Himanen, K. Image-Based Methods to Score Fungal Pathogen Symptom Progression and Severity in Excised Arabidopsis Leaves. Plants, 10, 158 (2021). doi:10.3390/plants10010158

dnormratio

Density Function of a Normal Ratio Distribution

Description

Density of the ratio of two independent Gaussian distributions.

Usage

```
dnormratio(z, bet, rho, delta)
```

Arguments

- z** length p numeric vector.
- bet, rho, delta** numeric values. The parameters (β, ρ, δ_y) of the distribution, see Details.

Details

Let two independant random variables $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$.

If we denote $\beta = \frac{\mu_x}{\mu_y}$, $\rho = \frac{\sigma_y}{\sigma_x}$ and $\delta_y = \frac{\sigma_y}{\mu_y}$, the probability distribution function of the ratio

$Z = \frac{X}{Y}$ is given by:

$$f_Z(z; \beta, \rho, \delta_y) = \frac{\rho}{\pi(1 + \rho^2 z^2)} \left[\exp\left(-\frac{\rho^2 \beta^2 + 1}{2\delta_y^2}\right) + \sqrt{\frac{\pi}{2}} q \operatorname{erf}\left(\frac{q}{\sqrt{2}}\right) \exp\left(-\frac{\rho^2(z - \beta)^2}{2\delta_y^2(1 + \rho^2 z^2)}\right) \right]$$

$$\text{with } q = \frac{1 + \beta \rho^2 z}{\delta_y \sqrt{1 + \rho^2 z^2}} \text{ and } \operatorname{erf}\left(\frac{q}{\sqrt{2}}\right) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{q}{\sqrt{2}}} \exp(-t^2) dt$$

Another expression of this density, used by the `estparnormratio()` function, is:

$$f_Z(z; \beta, \rho, \delta_y) = \frac{\rho}{\pi(1 + \rho^2 z^2)} \exp\left(-\frac{\rho^2 \beta^2 + 1}{2\delta_y^2}\right) {}_1F_1\left(1, \frac{1}{2}; \frac{1}{2\delta_y^2} \frac{(1 + \beta \rho^2 z)^2}{1 + \rho^2 z^2}\right)$$

where ${}_1F_1(a, b; x)$ is the confluent hypergeometric function (Kummer's function):

$${}_1F_1(a, b; x) = \sum_{n=0}^{+\infty} \frac{(a)_n}{(b)_n} \frac{x^n}{n!}$$

Value

Numeric: the value of density.

Author(s)

Pierre Santagostini, Ang  lina El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. *Remote Sensing* 15(2), 528 (2023). doi:10.3390/rs15020528

Marsaglia, G. 2006. Ratios of Normal Variables. *Journal of Statistical Software* 16. doi:10.18637/jss.v016.i04

D  az-Franc  s, E., Rubio, F.J., On the existence of a normal approximation to the distribution of the ratio of two independent normal random variables. *Stat Papers* 54, 309–323 (2013). doi:10.1007/s0036201204292

See Also

`pnormratio()`: probability distribution function.

`rnormratio()`: sample simulation.

`estparnormratio()`: parameter estimation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
dnormratio(0, bet = beta1, rho = rho1, delta = delta1)
dnormratio(0.5, bet = beta1, rho = rho1, delta = delta1)
curve(dnormratio(x, bet = beta1, rho = rho1, delta = delta1), from = -0.1, to = 0.7)

# Second example
beta2 <- 2
rho2 <- 2
delta2 <- 2
dnormratio(0, bet = beta2, rho = rho2, delta = delta2)
```

```
dnormratio(0.5, bet = beta2, rho = rho2, delta = delta2)
curve(dnormratio(x, bet = beta2, rho = rho2, delta = delta2), from = -0.1, to = 0.7)
```

estparnormratio

Estimation of the Parameters of a Normal Ratio Distribution

Description

Estimation of the parameters of a ratio $Z = \frac{X}{Y}$, X and Y being two independent random variables distributed according to Gaussian distributions, using the EM (estimation-maximization) algorithm or variational inference. Depending on the estimation method, the `estparnormratio` function calls `estparEM` (EM algorithm) or `estparVB` (variational Bayes).

Usage

```
estparnormratio(z, method = c("EM", "VB"), eps = 1e-06,
               display = FALSE, mux0 = 1, sigmax0 = 1,
               alphax0 = NULL, betax0 = NULL, muy0 = 1, sigmay0 = 1,
               alphay0 = NULL, betay0 = NULL)
```

```
estparEM(z, eps = 1e-06, display = FALSE, #plot = display,
        mux0 = 1, sigmax0 = 1, muy0 = 1, sigmay0 = 1)
```

```
estparVB(z, eps = 1e-06, display = FALSE, mux0 = 1, sigmax0 = 1,
        alphax0 = 1, betax0 = 1, muy0 = 1, sigmay0 = 1,
        alphay0 = 1, betay0 = 1)
```

```
estparEM(
  z,
  eps = 1e-06,
  display = FALSE,
  mux0 = 1,
  sigmax0 = 1,
  muy0 = 1,
  sigmay0 = 1
)
```

```
estparVB(
  z,
  eps = 1e-06,
  display = FALSE,
  mux0 = 1,
  sigmax0 = 1,
  alphax0 = 1,
  betax0 = 1,
  muy0 = 1,
```

```

    sigmay0 = 1,
    alphay0 = 1,
    betay0 = 1
)

```

Arguments

z numeric.

method the method used to estimate the parameters of the distribution. It can be "EM" (expectation-maximization) or "VB" (Variational Bayes).

eps numeric. Precision for the estimation of the parameters (see Details).

display logical. When TRUE the successive values of the stop criterion (distance between successive values) is printed.

mux0, sigmax0, muy0, sigmay0 initial values of the means and standard deviations of the X and Y variables. Default: mux0 = 1, sigmax0 = 1, muy0 = 1, sigmay0 = 1.

alphax0, betax0, alphay0, betay0 initial values for the variational Bayes method. Omitted if method="EM". If method="VB", if omitted, they are set to 1.

Details

Let a random variable: $Z = \frac{X}{Y}$,

X and Y being normally distributed: $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$.

The density probability of Z is:

$$f_Z(z; \beta, \rho, \delta_y) = \frac{\rho}{\pi(1 + \rho^2 z^2)} \exp\left(-\frac{\rho^2 \beta^2 + 1}{2\delta_y^2}\right) {}_1F_1\left(1, \frac{1}{2}; \frac{1}{2\delta_y^2} \frac{(1 + \beta \rho^2 z)^2}{1 + \rho^2 z^2}\right)$$

with: $\beta = \frac{\mu_x}{\mu_y}$, $\rho = \frac{\sigma_y}{\sigma_x}$, $\delta_y = \frac{\sigma_y}{\mu_y}$.

and ${}_1F_1(a, b; x)$ is the confluent hypergeometric function (Kummer's function):

$${}_1F_1(a, b; x) = \sum_{n=0}^{+\infty} \frac{(a)_n}{(b)_n} \frac{x^n}{n!}$$

If method = "EM", the means and standard deviations μ_x , σ_x , μ_y and σ_y are estimated with the EM algorithm, as presented in El Ghaziri et al. If method = "VB", they are estimated with the variational Bayes method as presented in Bouhlef et al.

Then the parameters β , ρ , δ_y of the Z distribution are computed from these means and standard deviations.

The estimation of μ_x , σ_x , μ_y and σ_y uses an iterative algorithm. The precision for their estimation is given by the eps parameter.

The computation uses the [kummer](#) function.

If there are ties in the z vector, it generates a warning, as there should be no ties in data distributed among a continuous distribution.

Value

A list of 3 elements beta, rho, delta: the estimated parameters of the Z distribution $\hat{\beta}, \hat{\rho}, \hat{\delta}_y$, with three attributes `attr(, "epsilon")` (precision of the result), `attr(, "k")` (number of iterations) and `attr(, "method")` (estimation method).

Author(s)

Pierre Santagostini, Angéline El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. *Remote Sensing* 15(2), 528 (2023). doi:10.3390/rs15020528

Bouhlel, N., Mercier, F., El Ghaziri, A., Rousseau, D., Parameter Estimation of the Normal Ratio Distribution with Variational Inference. 2023 31st European Signal Processing Conference (EU-SIPCO), Helsinki, Finland, 2023, pp. 1823-1827. doi:10.23919/EUSIPCO58844.2023.10290111

See Also

`dnormratio()`: probability density of a normal ratio.

`rnormratio()`: sample simulation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22

set.seed(1234)
z1 <- rnormratio(800, bet = beta1, rho = rho1, delta = delta1)

# With the EM method:
estparnormratio(z1, method = "EM")

# With the variational method:
estparnormratio(z1, method = "VB")

# Second example
beta2 <- 0.24
rho2 <- 4.21
delta2 <- 0.25

set.seed(1234)
z2 <- rnormratio(800, bet = beta2, rho = rho2, delta = delta2)

# With the EM method:
estparnormratio(z2, method = "EM")

# With the variational method:
```

```
estparnormratio(z2, method = "VB")
```

kummer

Confluent D-Hypergeometric Function

Description

Computes the Kummer's function, or confluent hypergeometric function.

Usage

```
kummer(a, b, z, eps = 1e-06)
```

Arguments

a	numeric.
b	numeric
z	numeric vector.
eps	numeric. Precision for the sum (default 1e-06).

Details

The Kummer's confluent hypergeometric function is given by:

$${}_1F_1(a, b; z) = \sum_{n=0}^{+\infty} \frac{(a)_n}{(b)_n} \frac{z^n}{n!}$$

where $(z)_p$ is the Pochhammer symbol (see [pochhammer](#)).

The eps argument gives the required precision for its computation. It is the `attr(, "epsilon")` attribute of the returned value.

Value

A numeric value: the value of the Kummer's function, with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Ang  lina El Ghaziri, Nizar Bouhlef

References

El Ghaziri, A., Bouhlef, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). [doi:10.3390/rs15020528](#)

Inpochhammer

*Logarithm of the Pochhammer Symbol***Description**

Computes the logarithm of the Pochhammer symbol.

Usage

Inpochhammer(x, n)

Arguments

x	numeric.
n	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)\dots(x+n-1)$$

So, if $n > 0$:

$$\log((x)_n) = \log(x) + \log(x+1) + \dots + \log(x+n-1)$$

If $n = 0$, $\log((x)_n) = \log(1) = 0$

Value

Numeric value. The logarithm of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

[pochhammer](#), [kummer](#)

Examples

```
Inpochhammer(2, 0)
Inpochhammer(2, 1)
Inpochhammer(2, 3)
```

pnormratio

*Cumulative Distribution of a Normal Ratio Distribution***Description**

Cumulative distribution of the ratio of two independent Gaussian distributions.

Usage

```
pnormratio(z, bet, rho, delta)
```

Arguments

`z` length p vector of quantiles.
`bet, rho, delta` numeric values. The parameters (β, ρ, δ_y) of the distribution, see Details.

Details

Let two independant random variables $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$.

If we denote $f_Z(z; \beta, \rho, \delta_y)$ the probability distribution function of the ratio $Z = \frac{X}{Y}$, with $\beta = \frac{\mu_x}{\mu_y}$, $\rho = \frac{\sigma_y}{\sigma_x}$ and $\delta_y = \frac{\sigma_y}{\mu_y}$ (see `dnormratio()`, Details section).

The probability distribution for Z is given by:

$$F(z; \beta, \rho, \delta_y) = \int_{-\infty}^z f_Z(z; \beta, \rho, \delta_y)$$

This integral is computed using numerical integration.

Value

Numeric: the value of density.

Author(s)

Pierre Santagostini, Ang  lina El Ghaziri, Nizar Bouhlel

References

- El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). [doi:10.3390/rs15020528](https://doi.org/10.3390/rs15020528)
- Marsaglia, G. 2006. Ratios of Normal Variables. Journal of Statistical Software 16. [doi:10.18637/jss.v016.i04](https://doi.org/10.18637/jss.v016.i04)
- D  az-Franc  s, E., Rubio, F.J., On the existence of a normal approximation to the distribution of the ratio of two independent normal random variables. Stat Papers 54, 309–323 (2013). [doi:10.1007/s0036201204292](https://doi.org/10.1007/s0036201204292)

See Also

`dnormratio()`: density function.
`rnormratio()`: sample simulation.
`estparnormratio()`: parameter estimation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
pnormratio(0, bet = beta1, rho = rho1, delta = delta1)
pnormratio(0.5, bet = beta1, rho = rho1, delta = delta1)
curve(pnormratio(x, bet = beta1, rho = rho1, delta = delta1), from = -0.1, to = 0.7)

# Second example
beta2 <- 2
rho2 <- 2
delta2 <- 2
pnormratio(0, bet = beta2, rho = rho2, delta = delta2)
pnormratio(0.5, bet = beta2, rho = rho2, delta = delta2)
curve(pnormratio(x, bet = beta2, rho = rho2, delta = delta2), from = -0.1, to = 0.7)
```

pochhammer

Pochhammer Symbol

Description

Computes the Pochhammer symbol.

Usage

```
pochhammer(x, n)
```

Arguments

`x` numeric.
`n` positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)\dots(x+n-1)$$

Value

Numeric value. The value of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

[lnpochhammer](#), [kummer](#)

Examples

```
pochhammer(2, 0)
pochhammer(2, 1)
pochhammer(2, 3)
```

rnormratio

Simulate from a Normal Ratio Distribution

Description

Simulate data from a ratio of two independent Gaussian distributions.

Usage

```
rnormratio(n, bet, rho, delta)
```

Arguments

n integer. Number of observations. If `length(n) > 1`, the length is taken to be the number required.

bet, rho, delta numeric values. The parameters (β, ρ, δ_y) of the distribution, see Details.

Details

Let two random variables $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$ with probability densities f_X and f_Y .

The parameters of the distribution of the ratio $Z = \frac{X}{Y}$ are: $\beta = \frac{\mu_x}{\mu_y}$, $\rho = \frac{\sigma_y}{\sigma_x}$, $\delta_y = \frac{\sigma_y}{\mu_y}$.

μ_x , σ_x , μ_y and σ_y are computed from β , ρ and δ_y (by fixing arbitrarily $\mu_x = 1$) and two random samples $(x_1, \dots, x_n)$ and $(y_1, \dots, y_n)$ are simulated.

Then $\left(\frac{x_1}{y_1}, \dots, \frac{x_n}{y_n}\right)$ is returned.

Value

A numeric vector: the produced sample.

Author(s)

Pierre Santagostini, Angéline El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. *Remote Sensing* 15(2), 528 (2023). doi:[10.3390/rs15020528](https://doi.org/10.3390/rs15020528)

Marsaglia, G. 2006. Ratios of Normal Variables. *Journal of Statistical Software* 16. doi:[10.18637/jss.v016.i04](https://doi.org/10.18637/jss.v016.i04)

Díaz-Francés, E., Rubio, F.J., On the existence of a normal approximation to the distribution of the ratio of two independent normal random variables. *Stat Papers* 54, 309–323 (2013). doi:[10.1007/s0036201204292](https://doi.org/10.1007/s0036201204292)

See Also

[dnormratio\(\)](#): probability density of a normal ratio.

[pnormratio\(\)](#): probability distribution function.

[estparnormratio\(\)](#): parameter estimation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
rnormratio(20, bet = beta1, rho = rho1, delta = delta1)

# Second example
beta2 <- 0.24
rho2 <- 4.21
delta2 <- 0.25
rnormratio(20, bet = beta2, rho = rho2, delta = delta2)
```

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